Short Papers

Spectral-Domain Analysis of Single and Coupled Cylindrical Striplines

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Abstract —A spectral-domain technique for finding the characteristic impedances of a single cylindrical stripline and a coupled pair of cylindrical striplines is presented. Assuming a charge distribution on the strip, the variational expression for the line capacitance for single cylindrical stripline is derived. Good agreement with published results is obtained. The cylindrical coupled strip and microstrip lines are also analyzed and a comparison with their planar counterparts is made.

I. INTRODUCTION

A cylindrical stripline consists of a circular arc strip placed between two cylindrical ground planes separated by dielectric layers. Recently, there has been a great deal of interest in analyzing cylindrical striplines [1]–[5]. Most of the interest is due to printed antennas on the cylindrical surfaces. They may also be useful as baluns, slotted lines, etc. [4].

Assuming that only TEM mode exists, Wang [1] and Joshi *et al.* [2] have solved the problem of finding the characteristic impedance of single cylindrical stripline with a homogeneous dielectric medium. Using conformal transformations, Joshi *et al.* [3] transformed a cylindrical stripline structure to a planar stripline with finite ground planes. Assuming that the strip width is small compared to 2π, the characteristic impedance of the resulting structure was found. Recently, Zeng *et al.* [4] also used conformal transformations to find the expressions for the characteristic impedance in a closed form for a cylindrical strip with zero and finite thickness. The authors have also found a closed-form expression for the characteristic impedance of a single stripline with multilayer dielectrics [5].

In the present work, the problem of a single and a coupled pair of cylindrical striplines is analyzed using spectral-domain techniques and TEM mode approximation. The problem is analyzed first by deriving the scalar Green's function for a unit charge located in the plane of cylindrical strip. By weighting the Green's function with the charge density and integrating over the strip, the scalar potential $\psi(\rho,\phi)$ is calculated at any point in the dielectric layers. From a knowledge of the potential function, a variational expression for capacitance and hence the characteristic impedance of a single cylindrical strip are obtained. The numerical results are compared with the results available in the literature [1], [2]. The method is then extended to determine even- and odd-mode characteristic impedances of a coupled pair of cylindrical striplines. The corresponding expressions for cylindrical microstrip lines are obtained by assuming that the outer cylinder is moved to infinity. The effect of warpage is investigated and numerical results are compared with published results.

Manuscript received September 29, 1986 revised February 11, 1987

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IEEE Log Number 8714789

II. THEORY

A. Single Stripline

The geometry of single cylindrical stripline is illustrated in Fig. 1(a) along with the notation to be used. Assuming only TEM mode propagation, scalar Green's function for regions I and II, $G^{\mathrm{I/II}}(\rho,\phi/\rho',\phi')$, satisfies the Laplace equation

$$\rho\frac{\partial}{\partial\rho}\left(\rho\frac{\partial G^{\mathrm{I/II}}}{\partial\rho}\right)+\frac{\partial^2 G^{\mathrm{I/II}}}{\partial\phi^2}=0. \tag{1}$$

It is easy to obtain the solution of (1) in the spectral domain. Assuming the solution of (1) in the form

$$G^{\mathrm{I/II}}(\rho,\phi)=\sum_{n=-\infty}^{\infty}\mathscr{G}^{\mathrm{I/II}}(\rho,n)e^{jn\phi} \tag{2}$$

where

$$\mathscr{G}^{\mathrm{I/II}}(\rho,n)=\frac{1}{2\pi}\int_0^{2\pi}G^{\mathrm{I/II}}(\rho,\phi)e^{-jn\phi}\,d\phi \tag{3}$$

and substituting (2) into (1) give

$$\rho^2\frac{\partial^2\mathscr{G}^{\mathrm{I/II}}}{\partial\rho^2}+\rho\frac{\partial\mathscr{G}^{\mathrm{I/II}}}{\partial\rho}-n^2\mathscr{G}^{\mathrm{I/II}}=0. \tag{4}$$

Applying proper boundary conditions, the solution of (4) is obtained as

$$\begin{aligned}\mathscr{G}^{\mathrm{I}}(\rho,n)&=a_0\ln(\rho/a) &&\text{for } n=0\\ &=A(n)\sinh(n\ln\rho/a) &&\text{for } n\neq 0\end{aligned} \tag{5}$$

$$\begin{aligned}\mathscr{G}^{\mathrm{II}}(\rho,n)&=a_0\frac{\ln(b/a)}{\ln(b/c)}\ln(\rho/c) &&\text{for } n=0\\ &=A(n)M_n\sinh(n\ln\rho/c) &&\text{for } n\neq 0.\end{aligned} \tag{6}$$

where

$$a_0=\frac{1}{2\pi\epsilon_0\left[\epsilon_{r_1}+\epsilon_{r_2}\dfrac{\ln(b/a)}{\ln(c/b)}\right]} \tag{7}$$

$$A(n)=\frac{1}{n2\pi\epsilon_0\left[\epsilon_{r_1}\coth(n\ln b/a)+\epsilon_{r_2}\coth(n\ln c/b)\right]\sinh(n\ln b/a)}$$

$$M_n=\frac{\sinh(n\ln b/a)}{\sinh(n\ln b/c)}. \tag{8}$$

Here, $\epsilon_{r1}=\epsilon_1/\epsilon_0$, $\epsilon_{r_2}=\epsilon_2/\epsilon_0$, and ϵ_0 = permittivity constant of free space.

From (5), (6), and (2), an expression for the scalar potential $\psi^{\mathrm{I/II}}(\rho,\phi)$ due to the charge distribution $q(b,\phi)$ on the strip is obtained [6]. The variational expression for the capacitance of the stripline is given by [6]

$$\frac{1}{C}=\frac{1}{Q^2}\int_0^{2\pi}\psi(b,\phi)\,q(b,\phi)\,b\,d\phi. \tag{9}$$

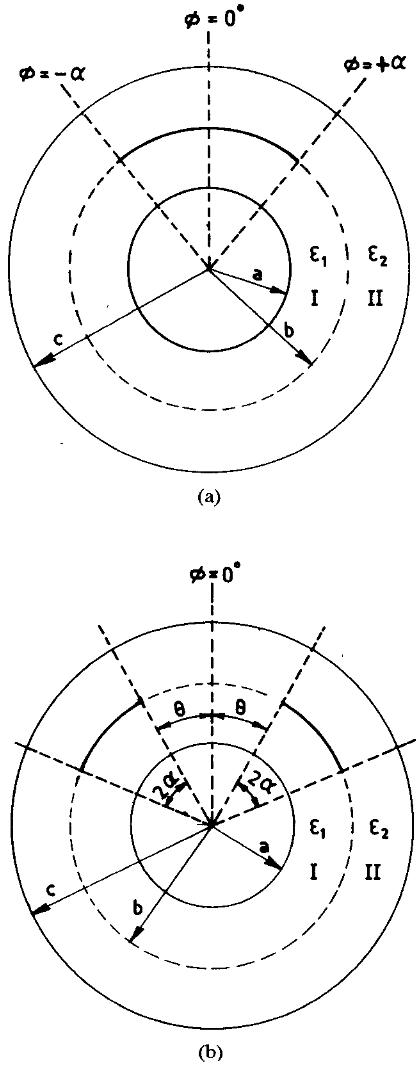

Fig. 1. (a) Cross section of single cylindrical stripline. (b) Cross section of a coupled pair of cylindrical striplines.

With simple mathematical manipulations, (9) becomes

$$\frac{\epsilon_0}{C}=\frac{\ln(b/a)}{2\pi\left[\epsilon_{r_1}+\epsilon_{r_2}\dfrac{\ln(b/a)}{\ln(c/b)}\right]}+\sum_{n=-\infty}^{\infty}\frac{2\pi[b|\tilde{q}(b,n)|/Q]^2(1-\delta_{n0})}{n\left[\epsilon_{r_1}\coth(n\ln(b/a))+\epsilon_{r_2}\coth(n\ln(c/b))\right]}\tag{10}$$

where

$$\delta_{n0}\begin{array}{ll}=1 & \text{for } n=0\\ =0 & \text{for } n\neq 0\end{array}$$

and

$$Q=\int_0^{2\pi}q(b,\phi)\,b\,d\phi$$

$$\tilde{q}(b,n)=\frac{1}{2\pi}\int_0^{2\pi}q(b,\phi)\,e^{-jn\phi}\,d\phi.$$

The characteristic impedance Z_0 of a single cylindrical stripline is then obtained as

$$Z_0=\frac{1}{v_0\sqrt{CC_a}}\tag{11}$$

where v_0 is the velocity of light in free space and C_a is the line capacitance with $\epsilon_{r_1}=\epsilon_{r_2}=1$.

B. Coupled Striplines

The geometry of the coupled cylindrical striplines is illustrated in Fig. 1(b) along with the notation to be used. Without loss of generality, the pair of strips may be assumed to be symmetrical about the $\phi=0°$ plane. Using the symmetry of the structure, it may be analyzed under even- and odd-mode excitations separately. Under the even-mode excitation, a magnetic wall at the $\phi=0°$ plane is assumed to be present. In the case of odd-mode excitation, an electric wall is assumed to be present at the $\phi=0°$ plane. In order to find the strip capacitances for structures under even- and odd-mode excitations, it is necessary to evaluate scalar potentials $\psi_{e/o}^{I/II}$ where the suffixes e and o stand for even and odd mode, respectively.

Following the steps described in the previous section for a single cylindrical strip, $\psi_{e/o}^{I/II}$ is obtained in the form

$$\psi_{e/o}^{I}=a_{0(e/o)}\ln(\rho/a)+\sum_{n=-\infty}^{\infty}(1-\delta_{n0})A_{e/o}(n)\sinh(n\ln\rho/a)\,e^{jn\phi}\tag{12}$$

$$\psi_{e/o}^{II}=a_{0(e/o)}\frac{\ln(b/a)}{\ln(b/c)}\ln(\rho/c)+\sum_{n=-\infty}^{\infty}(1-\delta_{n0})A_{e/o}(n)M_n\sinh(n\ln\rho/c)\,e^{jn\phi}.\tag{13}$$

In arriving at (12) and (13), the following extra boundary conditions are used:

$$H_{\rho e}^{I/II}(\rho,\phi)=0 \text{ at } \phi=0° \text{ for even-mode excitation}\tag{14}$$

$$E_{\rho o}^{I/II}(\rho,\phi)=0 \text{ at } \phi=0° \text{ for odd-mode excitation}\tag{15}$$

where $H_{\rho e}$ and $E_{\rho o}$ are tangential magnetic and electric fields, respectively, at the $\phi=0°$ plane.

In (12) and (13),

$$a_{0e}=a_0 \qquad a_{0o}=0\tag{16}$$

and

$$A_{e/o}(n)=2\pi b\tilde{q}_{e/o}(b,n)A(n)\tag{17}$$

where

$$q_e(b,n)=\frac{1}{\pi}\int_0^{\pi}q(b,\phi)\cos(n\phi)\,d\phi\tag{18}$$

$$q_o(b,n)=\frac{1}{\pi}\int_0^{\pi}q(b,\phi)\sin(n\phi)\,d\phi.\tag{19}$$

Using (12) and (13), the even- and odd-mode capacitances are obtained as

$$\frac{\epsilon_0}{2C_e}=\frac{\ln(b/a)}{2\pi\left[\epsilon_{r_1}+\epsilon_{r_2}\dfrac{\ln(b/a)}{\ln(c/b)}\right]}+\sum_{n=-\infty}^{\infty}\frac{2\pi[b|\tilde{q}_e(b,n)|/Q]^2(1-\delta_{n0})}{n\left[\epsilon_{r_1}\coth(n\ln b/a)+\epsilon_{r_2}\coth(n\ln c/b)\right]}\tag{20}$$

$$\frac{\epsilon_0}{2C_o}=\sum_{n=-\infty}^{\infty}\frac{2\pi[b|\tilde{q}_o(b,n)|/Q]^2(1-\delta_{n0})}{n\left[\epsilon_{r_1}\coth(n\ln(b/a)+\epsilon_{r_2}\coth(n\ln c/b)\right]}\tag{21}$$

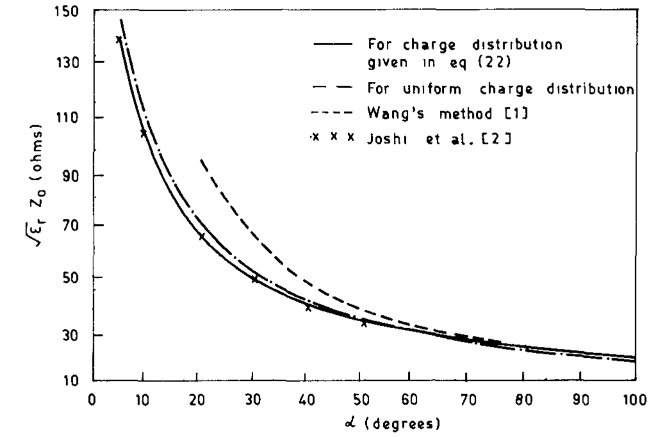

Fig. 2. Variation of characteristic impedance of cylindrical stripline with $c/a=2$, $b/a=1.4$, and $\epsilon_{r_1}=\epsilon_{r_2}=\epsilon_r$ as a function of half strip angle α.

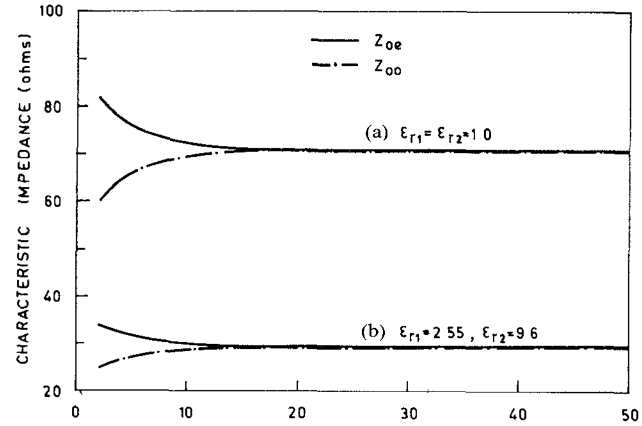

Fig. 3. Variation of even- and odd-mode impedances of a coupled pair of cylindrical striplines as a function of half separation angle θ with $c/a=1.5$, $b/a=1.2$, $\alpha=10°$. (a) $\epsilon_{r_1}=\epsilon_{r_2}=1.0$ (b) $\epsilon_{r_1}=2.55$, $\epsilon_{r_2}=9.6$.

The even- and odd-mode impedances Z_{0e} and Z_{0o} are then obtained by using (20), (21), and (11).

III. Results and Discussions

A. Single Stripline

In order to determine numerically the characteristic impedance of the cylindrical stripline, it is essential to know the charge distribution on the strip. Considering the field singularity at the strip edges, the trial charge distribution can be assumed to be of the form

$$q(\rho,\phi)=\frac{Q_0}{\sqrt{1-(\phi/\alpha)^2}}\delta(\rho-b) \qquad -\alpha\leqslant\phi\leqslant\alpha$$
$$=0 \qquad \text{otherwise} \tag{22}$$

where Q_0 is a constant.

Using the trial function given in (22), the characteristic impedance of the single cylindrical stripline with $c/a=2$, $b/a=1.4$, and $\epsilon_{r_1}=\epsilon_{r_2}=1$ is computed with the help of (10) and (11) and presented in Fig. 2 along with results obtained by Wang [1] and by Joshi *et al.* [2]. Good agreement with the results reported in [2] can be observed. As reported earlier in the literature [2], the results obtained by Wang [1] are inaccurate for low strip angles. It can also be seen from Fig. 2 that a uniform charge distribution over the strip also gives considerably accurate results for large strip angles.

B. Coupled Striplines

To obtain the numerical results for coupled cylindrical striplines, the trial charge distribution in (18) and (19) is assumed to be

$$q(\rho,\phi)=\frac{Q_0\delta(\rho-b)}{\sqrt{1-\{[\phi-(\theta+\alpha)]/\alpha\}^2}} \qquad \theta\leqslant\phi\leqslant\theta+2\alpha$$
$$=0 \qquad \text{otherwise.} \tag{23}$$

Using the expressions (20) and (21), the even- and odd-mode capacitances and hence the even- and odd-mode characteristic impedances Z_{0e} and Z_{0o} are computed for homogeneous ($\epsilon_{r_1}=\epsilon_{r_2}$) and inhomogeneous ($\epsilon_{r_1}=2.55$, $\epsilon_{r_2}=9.6$) cylindrical coupled striplines and presented in Fig. 3. For larger separation angles, it can be seen that Z_{0e} and Z_{0o} would be equal as the potential decays very fast outside the strip.

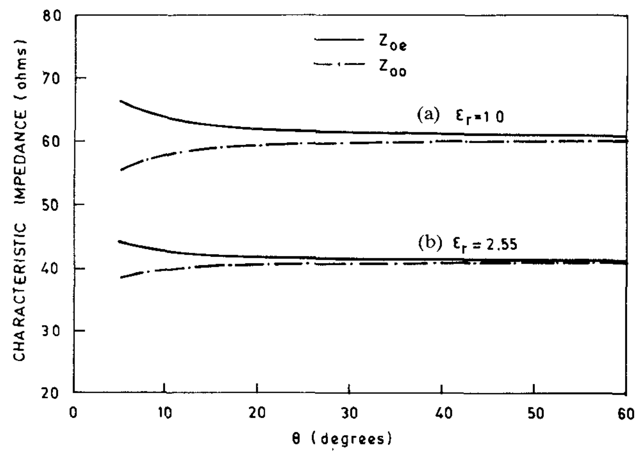

Fig. 4. Variation of even- and odd-mode impedances of coupled pair of cylindrical microstrip lines as a function of half separation angle θ with $b/a=1.1$ and $\alpha=10°$. (a) $\epsilon_r=1$. (b) $\epsilon_r=2.55$

C. Coupled Microstrip Lines

The expressions for coupled cylindrical microstrip lines can be obtained by assuming that $c\to\infty$ and $\epsilon_{r_2}=1$ while b and a remain finite. Thus, substituting $\coth(n\ln c/b)=1$ as $\ln c/a\to\infty$ in (20) and (21), the numerical results for even- and odd-mode characteristic impedances Z_{0e} and Z_{0o} are computed for coupled cylindrical microstrip lines and are presented in Fig. 4 for $\epsilon_{r1}=\epsilon_r=1.0$ and 2.55.

D. Warped Coupled Strip and Microstrip Lines

In order to study the effect of warpage, a structure with $c-b=b-a=h=h_1/2$, where $h_1=c-a$, is considered. For an angle 2α at the center, $W=2\alpha b$ and the separation between the two strips s subtending an angle 2θ at the center $s=2\theta b$. The even- and odd-mode characteristic impedances of warped coupled strip and microstrip lines can be obtained from the results of case (b) and case (c) by assuming that $a\to\infty$ while h_1 and h remain finite. The warpage of the line is indicated by the ratios h_1/a and h/a for the strip and microstrip cases, respectively. The numerical results for Z_{0e} and Z_{0o} of the warped case are computed and presented in Fig. 5 and compared with those of planar structure [7], [8]. It is observed that for a warpage of 0.01,

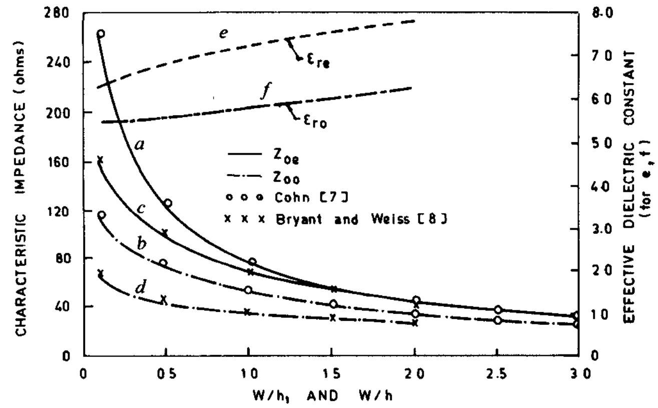

Fig. 5. Even- and odd-mode impedances as a function of W/h_1 and W/h for warped coupled strip and microstrip lines, respectively. Curves a and b represent coupled warped striplines with $\epsilon_{r_1} = \epsilon_{r_2} = 1$ and $s/h_1 = 0.1$. Curves c and d represent coupled warped microstrip lines with $\epsilon_r = 10$ and $s/h = 0.2$. Curves e and f represent the effective dielectric constants of coupled warped microstrip lines with $\epsilon_r = 10$ and $s/h = 0.2$.

the cylindrical structure approaches the case of a planar structure. Results for effective dielectric constants of even and odd modes ($\epsilon_{r(e/o)} = C_{(e/o)}/C_{(e/o)a}$) of coupled warped microstrip lines are also presented in Fig. 5.

IV. Conclusions

Spectral-domain techniques have been successfully applied for the case of cylindrical striplines and a coupled pair of cylindrical striplines. Using this technique, variational expressions for the line capacitances are derived for single and coupled cylindrical striplines. Using a suitable charge distribution on the strip, the characteristic impedances of single and coupled cylindrical striplines are obtained. The method could be successfully extended to coupled cylindrical microstrip lines and warped coupled strip and microstrip lines. Comparison with the results available in the literature is made wherever possible.

Acknowledgment

The authors wish to thank the reviewers for their suggestions.

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